RULING FRACTIONS

A Comprehensive Guide to Teaching Fractions with the Fraction SAFE-T Ruler®

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Introduction

The Fraction SAFE-T Ruler® is a triangular-shaped ruler with twelve measuring scales: twelfths, elevenths, tenths, ninths, eighths, sevenths, sixths, fifths, fourths, thirds, havles, and wholes. The Fraction SAFE-T Ruler® is two-units long. Each unit is ten centimeters (one decimeter) long. Fractions are measured compared to this unit. The tenths scale on the ruler may be used for metric measures and is compatible with other centimeter-based manipulatives.

Students in fourth through seventh grades use the Fraction SAFE-T Ruler® to visualize operations with fractions and decimals. Due to its resemblance to an engineering tool, the manipulative is also an appropriate tool for older students who are in need of remediation or reinforcement.

Recommended applications of the Fraction SAFE-T Ruler® include:

- comparing the relative size of fractions
- finding equivalent fractions
- using common-size fractions to find a common denominator
- adding and subtracting fractions
- connecting addition and subtraction of fractions to number line addition and subtraction of whole numbers
- multiplying and dividing fractions
- connecting multiplication of fractions to the formula for area
- deriving the algorithms for operations with fractions
- modeling operations with mixed numbers
- modeling operations with decimals using the tenths scale and the same process used for fractions
- estimating a decimal equivalent for a fraction

The teacher may use an overhead version of the Fraction SAFE-T Ruler® to demonstrate how to use this manipulative. Place a **Number Line Guide** on top of the transparency of the ruler and move it around and mark on it as needed. Each student, or pair of students will need a Fraction SAFE-T Ruler® and paper for sketching fraction lengths. You may choose to copy the Number Line Guides for them. For multiplying and dividing fractions, students will need several copies of **Unit Squares**. Note: Blackline Masters for the above items in bold are provided at the end of this guide and can be photocopied onto overhead transparencies.

Sample questions are provided for exploration by students individually or in pairs. Extension questions may be explored in pairs or small groups and then discussed as a whole class.

A linear model is used in this guide for modeling addition and subtraction of fractions because the dimension on measurement units stays the same when adding ($\frac{1}{2}$ foot 1 $\frac{1}{4}$ foot 5 $\frac{3}{4}$ foot). An area model is used for modeling multiplication and division of fractions because the dimension on measurement units becomes squared when multiplying ($\frac{1}{2}$ foot 3 $\frac{1}{4}$ foot 5 $\frac{1}{8}$ square feet). The area model also uses arrays (rows by columns) which help connect the model to the standard algorithms for multiplying and dividing fractions.

All of the scales on the Fraction SAFE-T Ruler® are marked as fractions of the One Whole scale, which is ten centimeters (or one decimeter) long.

Note: The illustrations in the manual are reduced in size.

Comparing Fractions

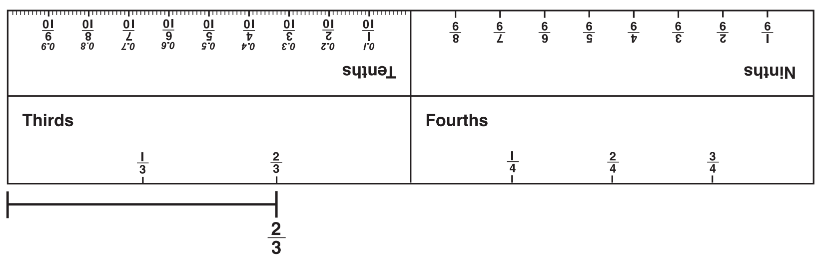
Materials:

One Fraction SAFE-T Ruler® per pair, paper, overhead projector, copy of number line guide for each student, Comparing Fractions student worksheet

Teacher Notes:

Compare $\frac{2}{3}$ to $\frac{2}{5}$.
On the overhead, model the above problem with students using these steps:

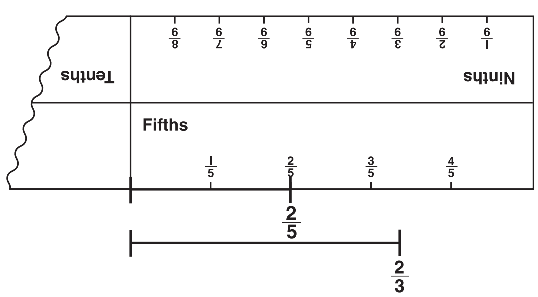
Step 1
Beginning at the top left edge of the *thirds* scale, draw a line that is $\frac{2}{3}$ units long.



Step 2
Mark both ends of the line with tick marks.

Step 3
Just above the $\frac{2}{3}$ unit line, align the left edge of the *fifths* scale with the left edge of the $\frac{2}{3}$ unit line. Draw a line that is $\frac{2}{5}$ units long.

Step 4
Mark both ends of the line and compare the two lines.
Students will see that the $\frac{2}{5}$ line is slightly shorter than the $\frac{2}{3}$ line.



Have the students compare the following fractions with a partner, modeling the same steps from your whole-class example.

$\frac{5}{8}$ and $\frac{5}{6}$

$\frac{4}{9}$ and $\frac{4}{11}$

$\frac{3}{5}$ and $\frac{3}{8}$

Class Discussion, Ideas, Prompts, and Questions:

What can you conclude about comparing fractions when only the denominators change?
Try out your ideas on these comparisons by predicting which fractions will be the larger fraction.

$\frac{5}{8}$ and $\frac{5}{12}$

$\frac{7}{10}$ and $\frac{7}{8}$

$\frac{2}{3}$ and $\frac{2}{7}$

Use your Fraction SAFE-T Ruler® to check your predictions.
Were your predictions correct? Explain why or why not.
Develop a rule for predicting which fraction will be larger/smaller when only the denominator changes.
The smaller the denominator, the larger each of the pieces the whole is divided into.

Sample Questions:

Use your Fraction SAFE-T Ruler® to solve the following problems:

1. Compare the lengths of these fractions

- **a.** $\frac{2}{6}$ and $\frac{3}{5}$
- **b.** $\frac{1}{2}$ and $\frac{3}{6}$

2. Name three fractions that are longer than $\frac{3}{4}$.

3. Name three fractions that are shorter than $\frac{1}{4}$.

Extension Questions:

Name two fractions that are the same length as each of the following fractions:

1. $\frac{2}{3}$

2. $\frac{1}{4}$

3. $\frac{3}{4}$

The answers to these questions are equivalent fractions and will be explored further in the next lesson.

Comparing Fractions

STUDENT WORKSHEET

1. What can you conclude about comparing fractions when only the denominators change?

2. Try out your ideas on these comparisons by predicting which fractions will be the larger fraction. Circle the fraction you think will be larger.

$\frac{5}{8}$ and $\frac{5}{12}$ 0

$\frac{7}{10}$ and $\frac{7}{8}$ 0

$\frac{2}{3}$ and $\frac{2}{7}$ 0

3. Use your Fraction SAFE-T Ruler® to check your predictions. Put a box around the fraction that is larger.

4. Were your predictions correct? Explain why or why not.

5. Develop a rule for predicting which fraction will be larger/smaller when only the denominator changes.

STUDENT NAME ______________________ **DATE** ____________

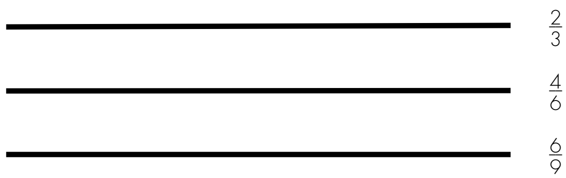
Equivalent Fractions

Materials:

One Fraction SAFE-T Ruler® per pair, paper, Equivalent Fractions Student Worksheet

Teacher Notes:

Equivalent fractions are fractions that are equal in length and value. For example, $\frac{2}{3}$ 5 $\frac{4}{6}$ 5 $\frac{6}{9}$



Explain to students that an algorithm is a standard step-by-step rule or set of rules to follow in order to reach a solution to a problem. It is important to tell students that there are many ways to derive an answer including an algorithm. When students are able to develop algorithms from their own work it makes sense to them, therefore they are more apt to understand and retain it for future use.

Class Discussion Ideas, Prompts, and Questions:

1. $\frac{1}{3}$ and $\frac{3}{9}$ **2.** $\frac{1}{4}$ and $\frac{3}{12}$ **3.** $\frac{2}{5}$ and $\frac{4}{10}$

What can you tell me about each pair of fractions?

The second numerator is a multiple of the first numerator (1 x 3 = 3), (1 x 3 = 3), and (2 x 2 = 4)

The second denominator is a multiple of the first denominator (3 x 3 = 9), (3 x 4= 12), and (2 x 5 = 10)

Both the numerator and denominator of the second fractions are the same multiples of the numerator and denominator of the first fractions.

What conclusions can be made from your observations?

To find an equivalent fraction, multiply the numerator and denominator by the same number.

Do you think you could find a different equivalent fraction for each set of fractions? Explain.

Yes, by multiplying the numerator and denominator by a different number.

Sample Questions:

Test your algorithm by finding at least two different equivalent fractions for each given fraction.

1. $\frac{3}{5}$ **2.** $\frac{5}{12}$ **3.** $\frac{7}{9}$

4. $\frac{2}{3}$ **5.** $\frac{1}{6}$ **6.** $\frac{1}{4}$

Check your answers using your Fraction SAFE-T Ruler® Can each answer be proven with your ruler? Explain why or why not.

No, because the Fraction SAFE-T Ruler® is limited to denominators up to 12.

Extension Questions:

Find three fractions the same length as the given fractions using your algorithm then check your answers with the Fraction SAFE-T Ruler®.

1. $\frac{2}{5}$

2. $\frac{1}{2}$

3. $\frac{5}{6}$

4. $\frac{6}{9}$

Extra Challenges

5. $\frac{5}{2}$

6. $1\frac{2}{3}$

Were you able to check all answers on the Fraction SAFE-T Ruler®? Explain.

How did your algorithm help you to solve each problem?

STUDENT WORKSHEET

Equivalent Fractions

$\frac{1}{3}$ and $\frac{3}{9}$

$\frac{1}{4}$ and $\frac{3}{12}$

$\frac{2}{5}$ and $\frac{4}{10}$

1. What can you tell me about each pair of fractions above?

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2. What conclusions can be made from your observations?

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3. Do you think you could find a different equivalent fraction for each set of fractions? Explain.

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4. Test your answer above by finding at least two different equivalent fractions for each given fraction.

a. $\frac{3}{5}$ 5 — 5 —   0

b. $\frac{5}{12}$ 5 — 5 —   0

c. $\frac{7}{9}$ 5 — 5 —   0

d. $\frac{2}{3}$ 5 — 5 —   0

e. $\frac{1}{6}$ 5 — 5 —   0

f. $\frac{1}{4}$ 5 — 5 —   0

Check your answers using your Fraction SAFE-T Ruler®. Can each answer be proven with your ruler? Explain why or why not.

STUDENT NAME ______________________________ **DATE** ____________

Simplifying Fractions

Materials:

One Fraction SAFE-T Ruler® per pair, paper, overhead projector

Teacher Notes:

Simplify $\frac{4}{8}$.

On the overhead, model the above problem with the students. Encourage students to realize that they must divide the numerator and denominator by the same number in order to simplify the fraction.

Find one equivalent fraction for $\frac{1}{3}$.

On the overhead, model the above problem with students. Encourage students to discover that in order to find an equivalent fraction, both the numerator and denominator should be multiplied by the same number.

Class Discussion Ideas, Prompts, and Questions:

Explain how simplifying a fraction and finding an equivalent fraction are similar.

Explain how they are different.

What conclusions can be made from your comparisons?

To find an equivalent fraction, you multiply both the numerator and denominator by the same number.

To simplify fractions, you divide both the numerator and denominator by the same number.

Sample Questions:

Try your algorithms on the following fractions.

Simplify these fractions:

1. $\frac{3}{6}$

2. $\frac{2}{4}$

3. $\frac{9}{12}$

4. $\frac{6}{8}$

5. $\frac{4}{2}$

Find one equivalent fraction for each fraction:

1. $\frac{2}{3}$

2. $\frac{5}{8}$

3. $\frac{1}{6}$

4. $\frac{3}{5}$

5. $\frac{1}{2}$

Adding Fractions

Materials:

One Fraction SAFE-T Ruler® per pair, paper, overhead projector, overhead Fraction SAFE-T Ruler®, and Adding Fractions Student Worksheets (pages 13, 14 & 15). Use adding machine tape for addition of mixed numbers.

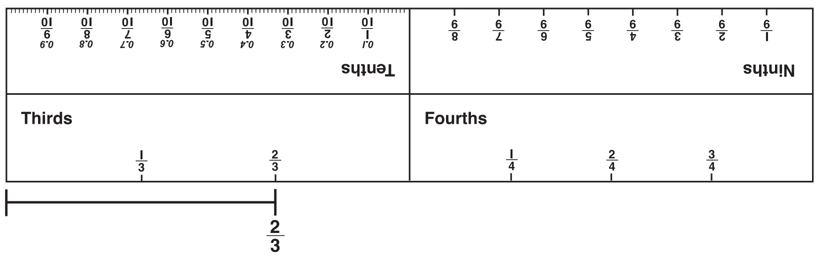
Teacher Notes:

$\frac{2}{3}$ 1 $\frac{1}{4}$

On the overhead, model the above problem with students using these steps:

Step 1

Beginning at the left edge of the *thirds* scale, draw a line that is $\frac{2}{3}$ units long.

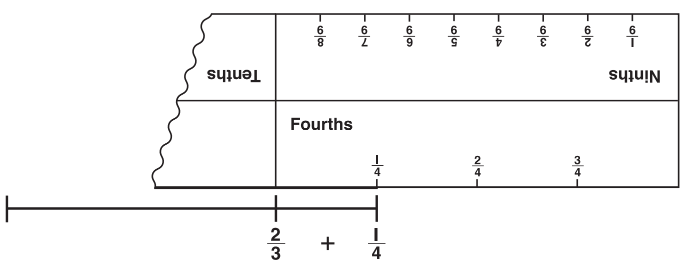


Step 2

Mark both ends of the line with tick marks.

Step 3

Draw and mark a new line $\frac{1}{4}$ units long beginning at the end of the $\frac{2}{3}$ unit line as shown in the illustration below. Mark the end of the $\frac{1}{4}$ units line with a tick mark.



Step 4

Find all scales that will evenly measure the total length of the two lines and give the answer. $\frac{11}{12}$

Class Discussion Ideas, Prompts, and Questions:

While the twelfths ruler is on the overhead ask students:
What do you notice about the relationship between the $\frac{2}{3}$ and twelfths? The $\frac{1}{4}$ and twelfths?

$\frac{2}{3}$ 5 $\frac{8}{12}$ $\frac{1}{4}$ 5 $\frac{3}{12}$

What can you conclude about how we arrived at $\frac{11}{12}$ for the answer? Added the numerators $\frac{8\ 1\ 3}{12}$ 5 $\frac{11}{12}$
Use the Fraction SAFE-T Ruler® to find all the scales that will evenly measure the answers to each of the expressions at the top of the following page. Be sure that **all** lengths match equally, no remainders allowed! You may work with a partner.

1. $\frac{1}{2}$ 1 $\frac{1}{4}$ **2.** $\frac{1}{3}$ 1 $\frac{2}{4}$ **3.** $\frac{2}{5}$ 1 $\frac{3}{10}$

Did any of the problems have more than one answer? Which one(s)?

Problems one and two.

How do the answers from problem one compare to one another?

All answers are equivalent fractions.

When comparing the answers from problem one, $\frac{3}{4}$, $\frac{6}{8}$, and $\frac{9}{12}$, what can you tell me about how each of the addends ($\frac{1}{2}$ or $\frac{1}{4}$) is related to the different results? Explain.

$\frac{1}{2}$ 5 $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{6}{12}$. $\frac{1}{4}$ 5 $\frac{1}{4}$, $\frac{2}{8}$, and $\frac{3}{12}$. Therefore, each addend can be changed into fractions with denominators of 4, 8, and 12.

How do you think each of the addends in problem two are related to their answers? Explain.

Same process as above.

Sample Questions:

List four equivalent fractions for each of the addends in the following problems then circle the first fraction for each addend with a common denominator: All equivalent fractions may not be found on your Fraction SAFE-T Ruler®.

The first problem has been shown for an example.

Example: $\frac{1}{2}$ 5 $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$

$\frac{1}{4}$ 5 $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, $\frac{5}{20}$

1. $\frac{1}{2}$ 1 $\frac{1}{4}$ **2.** $\frac{1}{2}$ 1 $\frac{2}{3}$ **3.** $\frac{1}{3}$ 1 $\frac{1}{6}$

4. $\frac{2}{5}$ 1 $\frac{1}{2}$ **5.** $\frac{1}{5}$ 1 $\frac{1}{3}$

Change each addend into equivalent fractions and add. Hint: Use the fractions you circled in the above exercise. Check your answers with your fraction ruler.

Were you able to check all results with your fraction ruler? Explain.

Explain why common denominators are important when adding fractions.

Use the common denominator with the largest scales to measure the sum of the fractions. What observations can you make about the numerators in the problems?

The numerators represent how many pieces there are. When the sizes of the pieces are equal, the numerators add to tell you the total number of pieces.

Extension: Would you get the same results if you use a different scale? Explain.

Explain how you can find common denominators without using your fraction ruler?

This is a good place to discuss the Least Common Multiple (LCM) method for finding common denominators of two or more fractions.

Find two fractions that do not have a common scale on the Fraction SAFE-T Ruler®.

Answers will vary — one example is $\frac{1}{3}$ and $\frac{1}{5}$.

Find the common denominator for your pair of fractions.

Explain the process used to find the common denominator.

Will this process work for all pairs of fractions? Explain.

Write a general rule for finding common denominators for two or more fractions.

Extension Questions:

Examine the following problem.
Anita and Michael solved the problem $\frac{1}{2}$ 1 $\frac{1}{12}$ 5 ? Anita got $\frac{4}{12}$ and Michael got $\frac{1}{3}$.
Is Anita's answer correct? Is Michael's answer correct? Explain how both answers can be correct.

How do you think we could avoid this type of confusion for other problems?

By simplifying all answers.

Review writing one whole as a fraction:
Using your Fraction SAFE-T Ruler® find as many one wholes as possible.

$\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, $\frac{7}{7}$, $\frac{8}{8}$, $\frac{9}{9}$, $\frac{10}{10}$, $\frac{11}{11}$, and $\frac{12}{12}$.

Adding fractions with sums greater than one whole. Find each sum.

1. $\frac{3}{4}$ 1 $\frac{1}{2}$ 5

2. $\frac{3}{10}$ 1 $\frac{4}{5}$ 5

3. $1\frac{2}{3}$ 1 $2\frac{5}{6}$ 5

The fraction ruler naturally leads to converting improper fractions to mixed fractions.

Encourage students to explore the commutative property in problem three by measuring 1 1 2 1 $\frac{2}{3}$ 1 $\frac{5}{6}$.
Compare the length to problem three. Explain.

What conclusions can be derived from your exploration?

Test your ideas by adding $1\frac{1}{4}$ 1 $1\frac{2}{3}$ two different ways.

Explain your results.

Adding Fractions

STUDENT WORKSHEET

Use the Fraction SAFE-T Ruler® to find all the scales that will evenly measure the answers to each of the expressions below. Be sure that **all** lengths match equally, no remainders allowed! You may work with a partner.

1. $\frac{1}{2}$ 1 $\frac{1}{4}$ 0

2. $\frac{1}{3}$ 1 $\frac{2}{4}$ 0

3. $\frac{2}{5}$ 1 $\frac{3}{10}$ 0

Did any of the problems have more than one answer? Which one(s)?

How do the answers from problem one compare to one another?

When comparing the answers from problem one, $\frac{3}{4}$, $\frac{6}{8}$, and $\frac{9}{12}$, what can you tell me about how each of the addends ($\frac{1}{2}$ or $\frac{1}{4}$) is related to the different results? Explain.

How do you think each of the addends in problem two are related to their answers? Explain.

List four equivalent fractions for each of the addends in the following problems then circle the first fraction for each addend with a common denominator: All equivalent fractions may not be found on your Fraction SAFE-T Ruler®.
The first problem has been shown for an example.

1. $\frac{1}{2}$ 1 $\frac{1}{4}$ 5 $\frac{3}{4}$ Example: $\frac{1}{2}$ 5 $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$

$\frac{1}{4}$ 5 $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, $\frac{5}{20}$

2. $\frac{1}{2}$ 1 $\frac{2}{3}$

3. $\frac{1}{3}$ 1 $\frac{1}{6}$

4. $\frac{2}{5}$ 1 $\frac{1}{2}$

5. $\frac{1}{5}$ 1 $\frac{1}{3}$

STUDENT NAME ______ **DATE** ______

Adding Fractions

Change each addend into equivalent fractions and add. Check your answers with your fraction ruler. Hint: Use the fractions you circled in the above exercise.

Were you able to check all results with your fraction ruler? Explain. ______________________

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Explain why common denominators are important when adding fractions. ______________________

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Use the common denominator with the largest scales to measure the sum of the fractions. What observations can you make about the numerators in the problems? ______________________

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Would you get the same results if you use a different scale? Explain. ______________________

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Explain how you can find common denominators without using your fraction ruler? ______________

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Find two fractions that do not have a common scale on the Fraction SAFE-T Ruler®.

____ ____

Find the common denominator for your pair of fractions. ______________________

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Explain the process used to find the common denominator. ______________________

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Will this process work for all pairs of fractions? Explain. ______________________

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Write a general rule for finding common denominators for two or more fractions. ______________

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STUDENT NAME ______________________ **DATE** ____________

Adding Fractions

STUDENT WORKSHEET

Examine the following problem.
Anita and Michael solved the problem $\frac{1}{2}$ 1 $\frac{1}{12}$ 5 ? Anita got $\frac{4}{12}$ and Michael got $\frac{1}{3}$.
Is Anita's answer correct? yes/no   Is Michael's answer correct? yes/no

Explain how both answers can be correct. ____

____

How do you think we could avoid this type of confusion for other problems? ____________________

____

Review writing one whole as a fraction:

Using your Fraction SAFE-T Ruler® find as many one wholes as possible.
$\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, $\frac{7}{7}$, $\frac{8}{8}$, $\frac{9}{9}$, $\frac{10}{10}$, $\frac{11}{11}$, and $\frac{12}{12}$.

Adding fractions with sums greater than one whole.

Find each sum.

1. $\frac{3}{4}$ 1 $\frac{1}{2}$ 5

2. $\frac{3}{10}$ 1 $\frac{4}{5}$ 5

3. $1\frac{2}{3}$ 1 $2\frac{5}{6}$ 5

Explore the commutative property in problem three by measuring 1 1 2 1 $\frac{2}{3}$ 1 $\frac{5}{6}$ Compare the length to problem three. Explain. ____

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What conclusions can be derived from your exploration? ________________________________

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Test your ideas by adding $1\frac{1}{4}$ 1 $1\frac{2}{3}$ two different ways. ______________________

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Explain your results. ____

____

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STUDENT NAME ________________________________ **DATE** ____________

Subtracting Fractions

Materials:

One Fraction SAFE-T Ruler® per pair, paper, overhead projector, overhead Fraction SAFE-T Rulers® and Subtracting Fractions Student Worksheet. Use adding machine tape when subtracting mixed fractions.

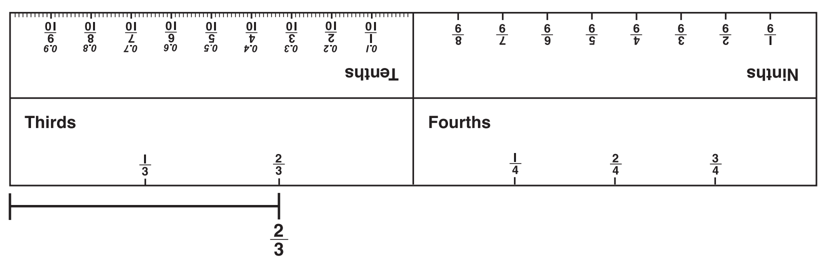
Teacher Notes:

$\frac{2}{3}$ 2 $\frac{1}{4}$

On the overhead, model the above problem with students using these steps:

Step 1

Beginning at the left edge of the *thirds* scale, draw a line that is $\frac{2}{3}$ units long.

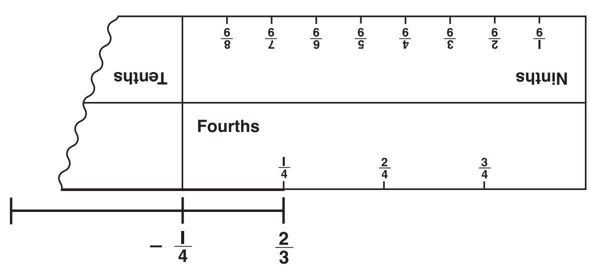


Step 2

Mark both ends of the line with tick marks.

Step 3

Subtract $\frac{1}{4}$ units long by measuring back (to the left) of the $\frac{2}{3}$ unit line as shown in the illustration below. Mark the end of the $\frac{1}{4}$ units line with a tick mark.

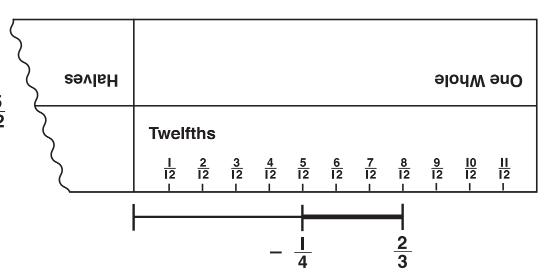


Step 4

Find all scales that will evenly measure the remaining length of the line and give the answer.

The remaining answer will be the portion of the line between where the $\frac{1}{4}$ line ended and the beginning of the $\frac{2}{3}$ line.

$\frac{2}{3} - \frac{1}{4} = \frac{5}{12}$



Class Discussion Ideas, Prompts, and Questions:

Explain how you chose which scale to measure each remainder.

Compare the steps for adding fractions to those for subtracting fractions be sure to include:

This would be a good place to review denominator (the number of equal parts the whole is divided into), and numerator (the number of equal parts of the whole being referred to).

How is the algorithm the same for both adding and subtracting fractions? How is it different?

Use the words that link the Fraction SAFE-T Ruler® to the algorithm such as common scale on the ruler and common denominator in the algorithm.

Is a common denominator necessary in order to subtract fractions? Explain.

Explain what it means to regroup fractions.

Describe how to regroup/borrow from a whole when subtracting fractions.

The Fraction SAFE-T Ruler® helps students to understand the concept of borrowing one whole as a fraction rather than a 10 as is done with whole numbers.

Sample Questions:

Use your Fraction SAFE-T Ruler® to solve these problems.
Use the largest common scale on the Fraction SAFE-T Ruler® to measure the result. Simplify all answers.

1. $\frac{1}{2}$ 2 $\frac{1}{3}$ 5

2. $\frac{2}{3}$ 2 $\frac{1}{4}$ 5

3. 1 2 $\frac{2}{5}$ 5

4. $\frac{5}{6}$ 2 $\frac{1}{6}$ 5

Extension Questions:

The following problems involve mixed fractions. Students should be encouraged to measure the wholes with the whole scale. When students subtract, they are left with an unmarked scale.

1. $1\frac{2}{7}$ 2 $\frac{4}{7}$ 5

2. $2\frac{3}{7}$ 2 $1\frac{6}{7}$ 5

3. $2\frac{1}{6}$ 2 $\frac{1}{3}$ 5

4. 3 2 $1\frac{8}{11}$ 5

STUDENT WORKSHEET

Subtracting Fractions

Use your Fraction SAFE-T Ruler® to solve these problems.
Use the largest common scale on the Fraction SAFE-T Ruler® to measure the result. Simplify all answers.

1. $\frac{1}{2}$ 2 $\frac{1}{3}$ 5

0

2. $\frac{2}{3}$ 2 $\frac{1}{4}$ 5

0

3. 1 2 $\frac{2}{5}$ 5

0

4. $\frac{5}{6}$ 2 $\frac{1}{6}$ 5

0

The following problems involve mixed fractions. Measure the wholes with the whole scale.

1. $1\frac{2}{7}$ 2 $\frac{4}{7}$ 5

2. $2\frac{3}{7}$ 2 $1\frac{6}{7}$ 5

3. $2\frac{1}{6}$ 2 $\frac{1}{3}$ 5

4. 3 2 $1\frac{8}{11}$ 5

STUDENT NAME ____ **DATE** _______________

Multiplying Fractions

Materials:

One Fraction SAFE-T Ruler® per pair, several copies of the Unit Square blackline master for each activity

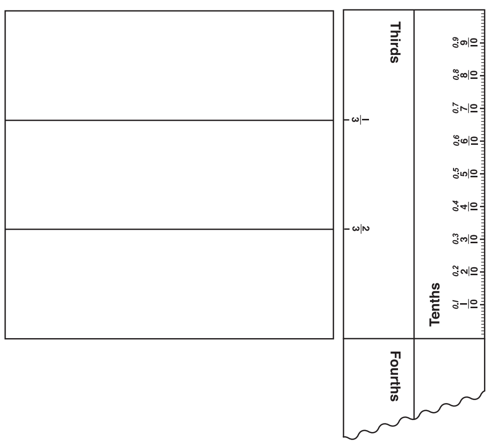
Teacher Notes:

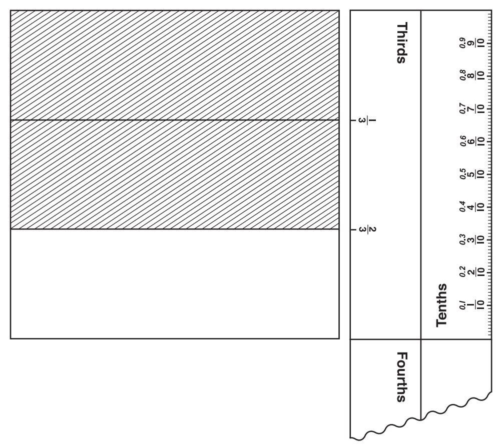
When introducing multiplication of fractions students need to understand what it means to multiply. For example, 1 3 4 means 1 of a group of 4, or 4. $\frac{2}{3}$ 3 $\frac{4}{5}$ means $\frac{2}{3}$'s of a group of $\frac{4}{5}$'s. To understand what is happening with fractions it is necessary to begin with the whole, therefore pairs of students will need several copies of the Unit Square.
When multiplying use the unit square then follow these easy steps:

Step 1

$\frac{2}{3}$ 3 $\frac{4}{5}$ 5 ______

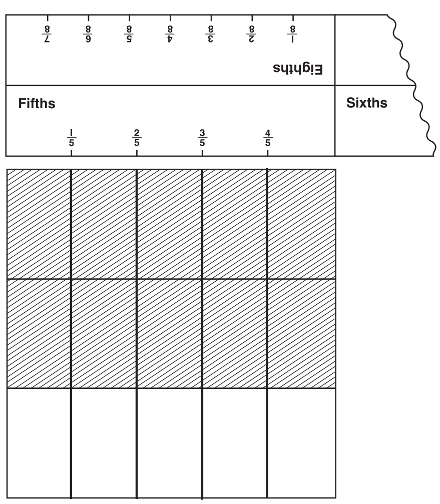
Using your Fraction SAFE-T Ruler®, divide the square horizontally by the number of sections indicated by the denominator of the first fraction, in this example by thirds. Shade two of the thirds, as shown in the illustration below.

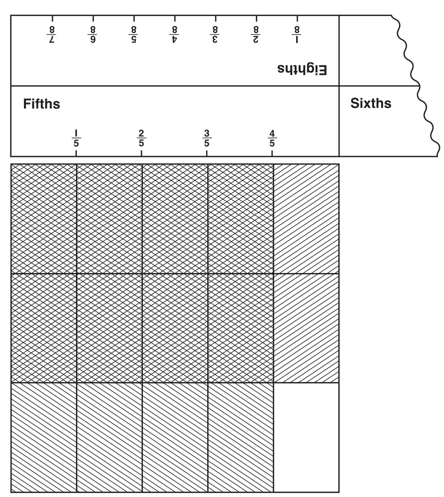




Step 2

Using your Fraction SAFE-T Ruler®, divide the square vertically by the number of sections indicated by the denominator of the second fraction, in this example by fifths. Shade four of the fifths, as shown in the illustration below.





Class Discussion Ideas, Prompts, and Questions:

How many rectangles is the unit square divided into?

Fifteen

How many of the fifteen rectangles are double shaded?

Eight of the fifteen or $\frac{8}{15}$

What can be concluded by this observation?

That $\frac{2}{3}$ 3 $\frac{4}{5}$ 5 $\frac{8}{15}$

Examine the final array.

What relationships can you find between the array and the fractions?

The whole array is divided into 15 parts or 3 3 *5 where 3 is the denominator of the first fraction and 5 the denominator of the second fraction. The double shading forms a second array of 8. Eight is equal to 2* 3 *4. Two is the numerator of the first fraction and 4 is the numerator of the second.*

Sample Questions:

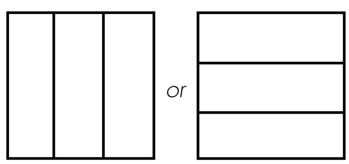
Try your ideas on these fractions:

1. $\frac{1}{2}$ 3 $\frac{1}{3}$ 5 —— **2.** $\frac{1}{4}$ 3 $\frac{1}{5}$ 5 —— **3.** $\frac{1}{3}$ 3 $\frac{1}{6}$ 5 ——

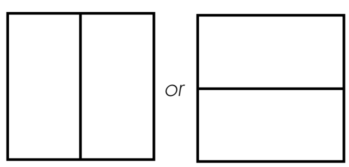
Use unit squares to prove your answers.

Exploring problem one:

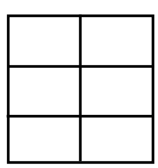
How many equal sections is the unit divided into when divided into thirds? *3*



How many equal sections is the unit divided into when divided into halves? *2*



When the unit is divided into thirds one way and halves the other, how many equal sections will the unit be divided into? *6*



From your observations what can you tell me about how the results of $\frac{1}{2}$ 3 $\frac{1}{3}$ and $\frac{1}{3}$ 3 $\frac{1}{2}$ are related?

The number of equal sections is 6 in both cases.

Extension Questions:

Explain how you would determine the number of pieces any whole is divided into when two fractions are multiples. Sketch examples to support your explanation.

Consider the whole number multiplication problem 6 3 3 5 18 and the fractional multiplication problem $\frac{1}{6}$ 3 $\frac{1}{3}$ 5 $\frac{1}{18}$. Explain how these two problems are similar and different. Support your explanation with sketches.

What conclusions can be made concerning the multiplication of fractions? Write your conclusions in the form of an algorithm.

Use your conclusions to solve the following examples. Check your answers by following the steps for multiplying fractions using the unit square.

$\frac{1}{4}$ 3 $\frac{2}{3}$ 5 ____

$\frac{2}{5}$ 3 $\frac{3}{4}$ 5 ____

Explain how you would tell someone to solve the following problem without using the unit square. Write your explanation as a step-by-step rule. $\frac{1}{2}$ 3 $\frac{3}{4}$ 5 ____

Exchange your rule with your partner. Can your partner solve the problem using your rule? Why or why not? If not, what can be changed?

These fractions include the effect of the numerator to finish developing the multiplication algorithm.

1. $\frac{4}{5}$ 3 $\frac{2}{3}$ 5 ____

2. $\frac{3}{4}$ 3 $\frac{3}{7}$ 5 ____

3. $\frac{1}{2}$ 3 $\frac{2}{3}$ 5 ____

Look at your sketch for problem one. How many of the fifths are shaded? *four* How many of the thirds are shaded? *two*

How many pieces are shaded twice? *eight*

What whole number multiplication problem is this related to? 4 3 2 5 8

This is a multiplication array for the double shaded part of the unit square.

How could you find the number of pieces that will be double shaded when two fractions are multiplied (use fractions with any numerator)? Sketch examples to support your theory.

Multiply the numerators. Sketches may vary.

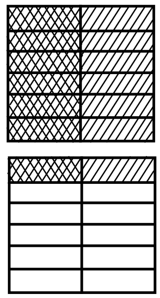
Use what you have learned from both sets of questions to write a rule for multiplying two fractions.

Multiply the numerators and multiply the denominators. (Remind students to check to see if the result can be simplified).

Test your rule using fractions you can measure with your Fraction SAFE-T Ruler®. Check your result by sketching the fractions with your ruler.

More Extension Questions:

The following problems include mixed fractions and the need for multiple unit squares. Discuss with the class the meaning of $\frac{6}{5}$ (as in problem one below) of a unit square and how to sketch it. A mixed fraction will result in pieces of different sizes unless students divide the wholes to match the fraction associated with them. This will connect the model to the algorithmic step of multiplying each whole number by its respective fraction's denominator. Circulate as students work through these examples. Ask questions to help them discover the adjustments needed for mixed fractions.

You should see a sketch like this  for problem one. If not, ask the students how many pieces are in one whole.

The number of divisions in all unit squares should match each other and the denominator, before it is reduced. Or visually, all the pieces need to be the same size.

For problems 2, 3, and 4, after a sketch is correctly made, ask what the sketch shows about writing the mixed number as a fraction.

Answer: ($1\frac{1}{2}$ 5 $\frac{3}{2}$ and 4 5 $\frac{4}{1}$)

Follow this up by asking what the 1 in the denominator means.

A piece that is the size of a whole.

1. $\frac{6}{5}$ 3 $\frac{1}{2}$ 5

2. $\frac{2}{5}$ 3 $1\frac{1}{2}$ 5

3. $1\frac{1}{2}$ 3 $1\frac{1}{3}$ 5

4. $\frac{3}{4}$ 3 4 5

5. What new rules are needed for multiplying mixed fractions?

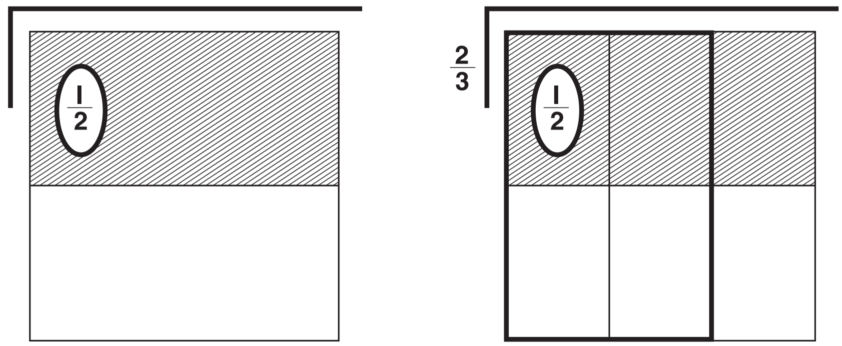
The wholes must be divided into the same number of pieces as the fraction so all the pieces are the same size. Once students have discovered this visually you might want to convert the rule to mathematical terms such as "the mixed fractions must be converted to improper fractions" before multiplying the numerators and denominators.

Dividing Fractions

Teacher Notes:

$\frac{1}{2}$ 4 $\frac{2}{3}$

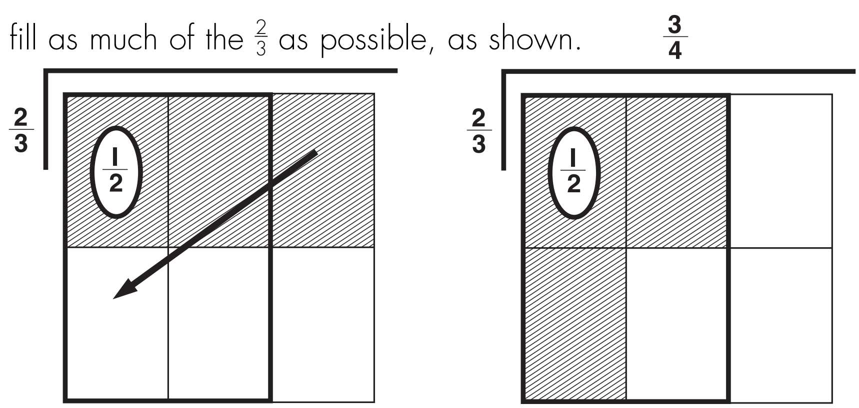
Think — "What is $\frac{1}{2}$ shared among $\frac{2}{3}$? $\frac{1}{2}$ does not even fill $\frac{2}{3}$ so think — What part of $\frac{2}{3}$ will $\frac{1}{2}$ fill?"
Divide the unit square horizontally into halves using the Fraction SAFE-T Ruler® as before and shade one of them. Just as in multiplication the first number in the operation is drawn first with horizontal lines. Divide the unit square vertically into thirds and outline two of them. This outlined part now represents the *whole part we are trying to fill*. Notice that the whole is no longer represented by the unit square. The outlining is done to emphasize this.



The array for the whole $\frac{2}{3}$ is divided into four pieces as a **2** 3 **2** array. The array for the whole is shown in the denominator as it was for multiplication 2 3 2. The amount shaded is a **1** 3 **3** array. The array for the shaded part is shown in the numerator as it was for multiplication. 1 3 $\frac{3}{2}$ 3 2 This matches the invert and multiply algorithm.

$\frac{1}{2}$ 4 $\frac{2}{3}$ 5 $\frac{1}{2}$ 3 $\frac{3}{2}$ 5 $\frac{1\ 3\ 3}{2\ 3\ 2}$

Move the shaded piece to fill as much of the $\frac{2}{3}$ as possible, as shown.



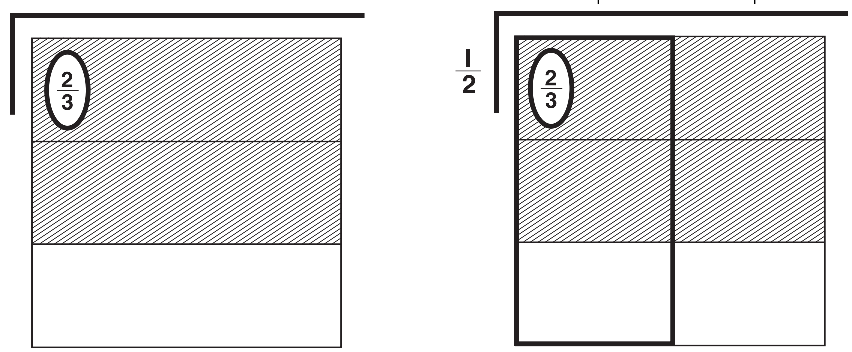
$\frac{1}{2}$ fills $\frac{3}{4}$ of $\frac{2}{3}$. $\frac{1\ 3\ 3}{2\ 3\ 2}$ 5 $\frac{3}{4}$

Remember, when multiplying, a "whole" refers to one unit square. When dividing, the meaning of a "whole" is defined by the divisor.

Class Discussion Ideas, Prompts, and Questions:

$\frac{2}{3}$ 4 $\frac{1}{2}$ The answer to this is a mixed number and illustrates that division of fractions is not commutative. Think — "What is $\frac{2}{3}$ shared among $\frac{1}{2}$?" $\frac{2}{3}$ will fill more than a whole $\frac{1}{2}$ so think —
"How many $\frac{1}{2}$'s will $\frac{2}{3}$ fill?"

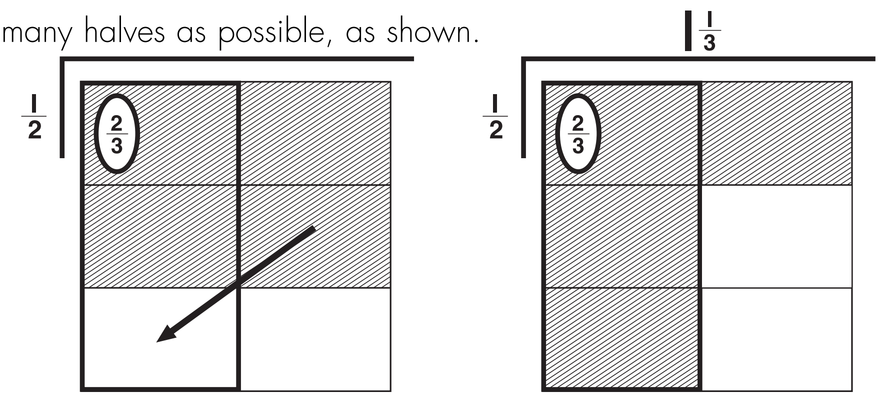
Divide the unit square horizontally into thirds using the fraction ruler as before, and shade two of them. Divide the unit square vertically into halves and outline one of them. This outlined part now represents *the whole part we are trying to fill.*



The array for the whole $\frac{1}{2}$ is divided into three pieces as a **3** 3 **1** array. The array for the whole is shown in the denominator $\frac{}{3\;3\;1}$. The amount shaded is a **2** 3 **2** array. The array for the shaded part is shown in the numerator. $\frac{2\;3\;2}{3\;3\;1}$ This matches the invert and multiply algorithm.

$\frac{2}{3}$ 4 $\frac{1}{2}$ 5 $\frac{2}{3}$ 3 $\frac{2}{1}$ 5 $\frac{2\;3\;2}{3\;3\;1}$

Move the shaded piece to fill as many halves as possible, as shown.



$\frac{2}{3}$ fills $1\frac{1}{3}$ halves. $\frac{2\;3\;2}{3\;3\;1}$ 5 $\frac{4}{3}$ 5 $1\frac{1}{3}$ The picture illustrates the conversion from $\frac{4}{3}$ 5 $1\frac{1}{3}$.

Remember, when dividing, the meaning of a "whole" is defined by the divisor.

*A good teacher demonstration or student exploration can be done with standard liquid (clear) and dry (opaque) measuring cups. Fill a liquid measuring cup to the $\frac{2}{3}$ cup mark. (For a teacher demonstration, dyed water shows up well.) Tell the class that this needs to be shared among half cups. The question is "How many $\frac{1}{2}$'s will $\frac{2}{3}$ fill?" Pour the $\frac{2}{3}$ cup of liquid into the dry $\frac{1}{2}$ cup measuring cup until the **whole** $\frac{1}{2}$ cup is full. This is one $\frac{1}{2}$ cup. Pour the remaining liquid into another dry $\frac{1}{2}$ cup measuring cup and ask a few students to estimate what fraction of the $\frac{1}{2}$ cup is full. This demonstration helps students understand why the divisor defines the **whole** as a fraction.*

Sample Questions:

1. $\frac{2}{5}$ 4 $\frac{2}{3}$ 5 **2.** $\frac{3}{5}$ 4 $\frac{7}{10}$ 5 **3.** $1\frac{1}{2}$ 4 $\frac{1}{2}$ 5

4. When you divide whole numbers the answer is always smaller than the dividend. (18 ÷ 3 = 6; 6 is smaller than 18). When you divide by a fraction less than one, is the answer larger or smaller than the dividend? Give an example and explain your reasoning with sketches or words.

The answer is larger. If things are divided into parts there are more parts. For example: 6 cookies divided into groups of 2 cookies means 3 children can have cookies. 6 cookies divided into groups of half cookies means 12 children can have a piece of cookie.

Operations with Decimals

Teacher Notes:

The tenths scale on the Fraction SAFE-T Ruler® is marked in decimal notation as well as fractional notation. Sample questions alternate between tenths and decimal notation to help students make the connection between fractions and decimals. The measuring for operations on decimals with the ruler is the same as it would be for measuring tenths.

Sample Questions:

1. $\frac{3}{10}$ 1 $\frac{6}{10}$ 5

2. 0.4 1 0.2 5

3. $1\frac{3}{10}$ 2 $\frac{7}{10}$ 5

4. 1.2 2 0.8 5

5. $\frac{3}{10}$ 3 $\frac{5}{10}$ 5

6. 0.6 3 0.8 5

7. $1\frac{4}{10}$ 4 $\frac{7}{10}$ 5

8. 1.5 4 0.5 5

9. 2 3 0.4 5

10. 2 3 0.5 5

Extension Questions:

1. When you add tenths to tenths how many decimal places are in the answer? Give an example.

One, although it may be filled with a zero. Examples: 0.5 1 *0.7* 5 *1.2   0.4* 1 *0.6* 5 *1.0   (Note to teachers: 1.0 is equivalent to 1 in value and you may encourage your students to write it either way. In the sciences, the difference shows the precision in the measurement.)*

2. When you multiply tenths by tenths how many decimal places are in the answer? Explain and give an example.

Two, although either place may be filled with a zero (see examples below). A ten by ten array creates one hundred pieces, so each piece is one hundredth (0.01) of a whole. If your students do not share each of the cases below in the discussion, you may want to talk about them. Examples: 0.3 3 *0.4* 5 *0.12   0.4* 3 *0.5* 5 *0.20   0.3* 3 *0.2* 5 *0.06*

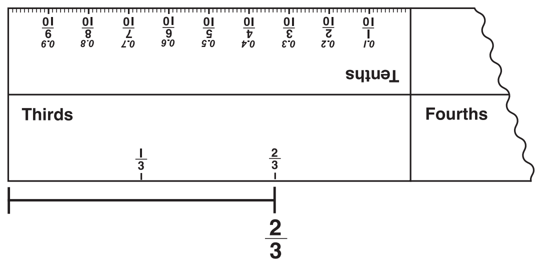
3. When you multiply a whole by a tenth how many decimal places are in the answer? Explain and give an example.

One, although it may be filled with a zero. A one by ten array creates ten places, so each piece is one tenth (0.1) of a whole. Examples: 3 3 *0.6* 5 *1.8   5* 3 *0.6* 5 *3.0*

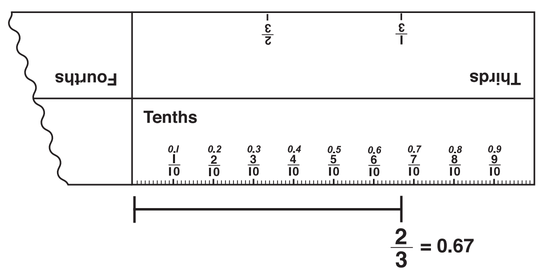
Decimal Equivalents of Fractional Values

Teacher Notes:

Measure and mark the length of the fraction $\frac{2}{3}$ with the Fraction SAFE-T Ruler®.



Measure it again with the decimal *(tenths)* scale, estimating the decimal value to the hundredths place when necessary.



Sample Questions:

Estimate the decimal equivalent for each fraction by measuring with the fraction ruler.

1. $\frac{1}{2}$ 5

2. $\frac{1}{3}$ 5

3. $\frac{1}{4}$ 5

4. $\frac{1}{5}$ 5

5. $\frac{2}{5}$ 5

6. $\frac{3}{4}$ 5

Extension Questions:

1. Which fractions can be measured exactly with tenths? Explain.

Halves and fifths can because two and five are factors of ten.

2. Which fractions can be measured exactly with hundredths? Explain. (Hint: Thinking about fractions of a dollar help when thinking about hundredths.)

Halves, fourths, and fifths can because two, four, and five are factors of one hundred.

Notes

Blackline Masters

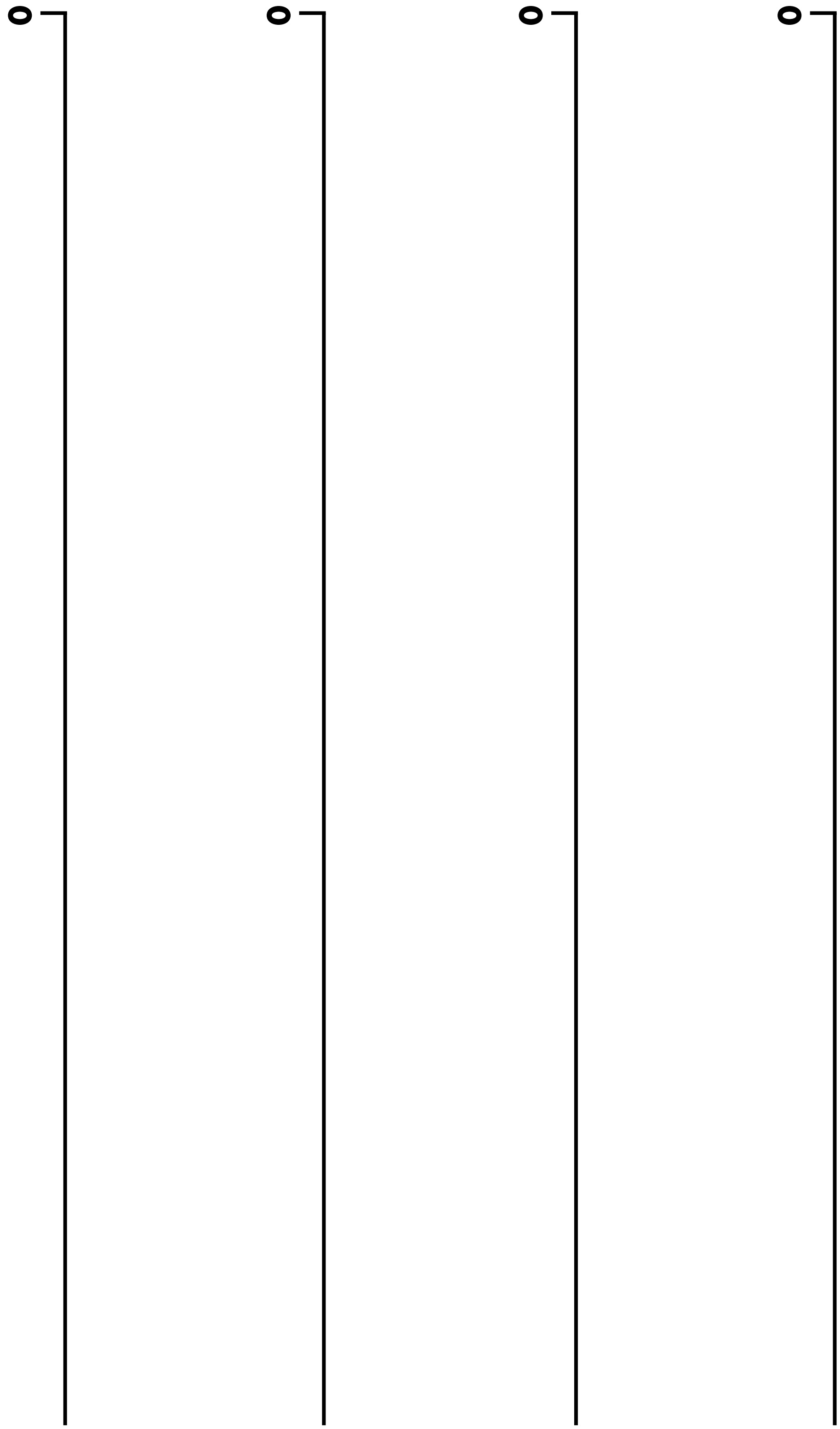
• Number Line Guide for the overhead and/or students

• Unit Squares for the overhead and/or students

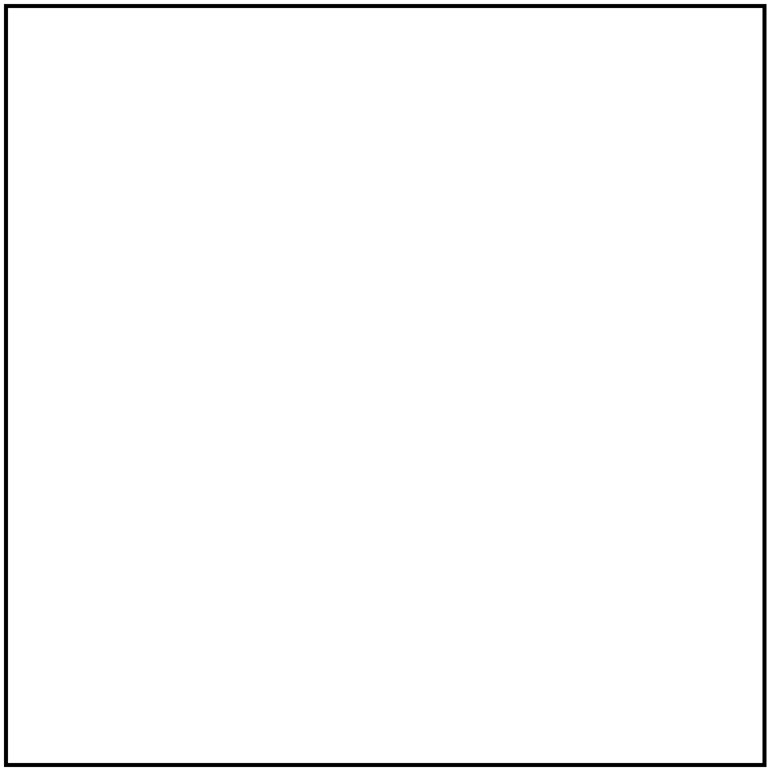
• Fraction SAFE-T Ruler® for the overhead

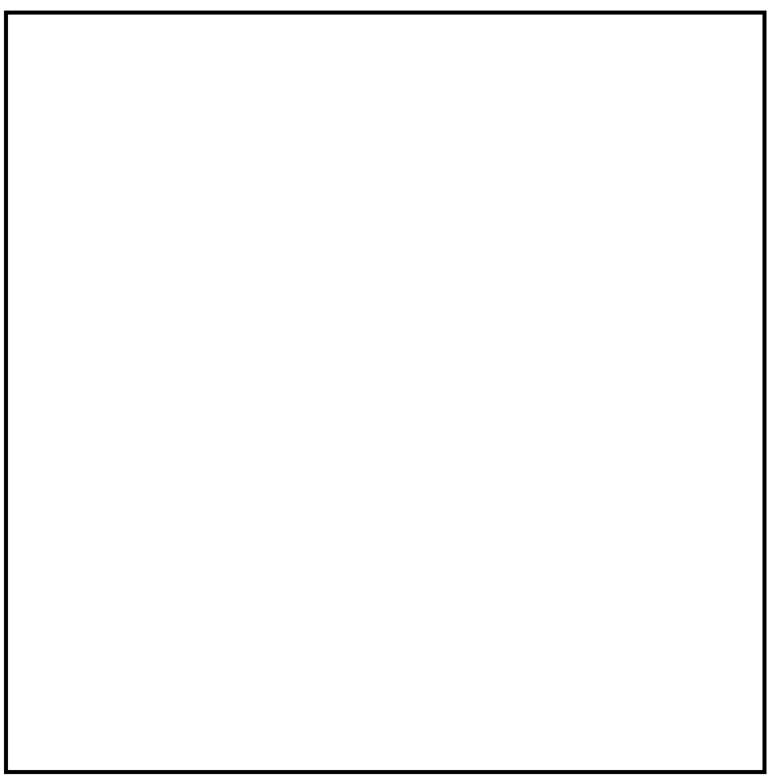
RULING FRACTIONS
A Comprehensive Guide to Teaching
Fractions with the Fraction SAFE-T Ruler®

Number Line Guide

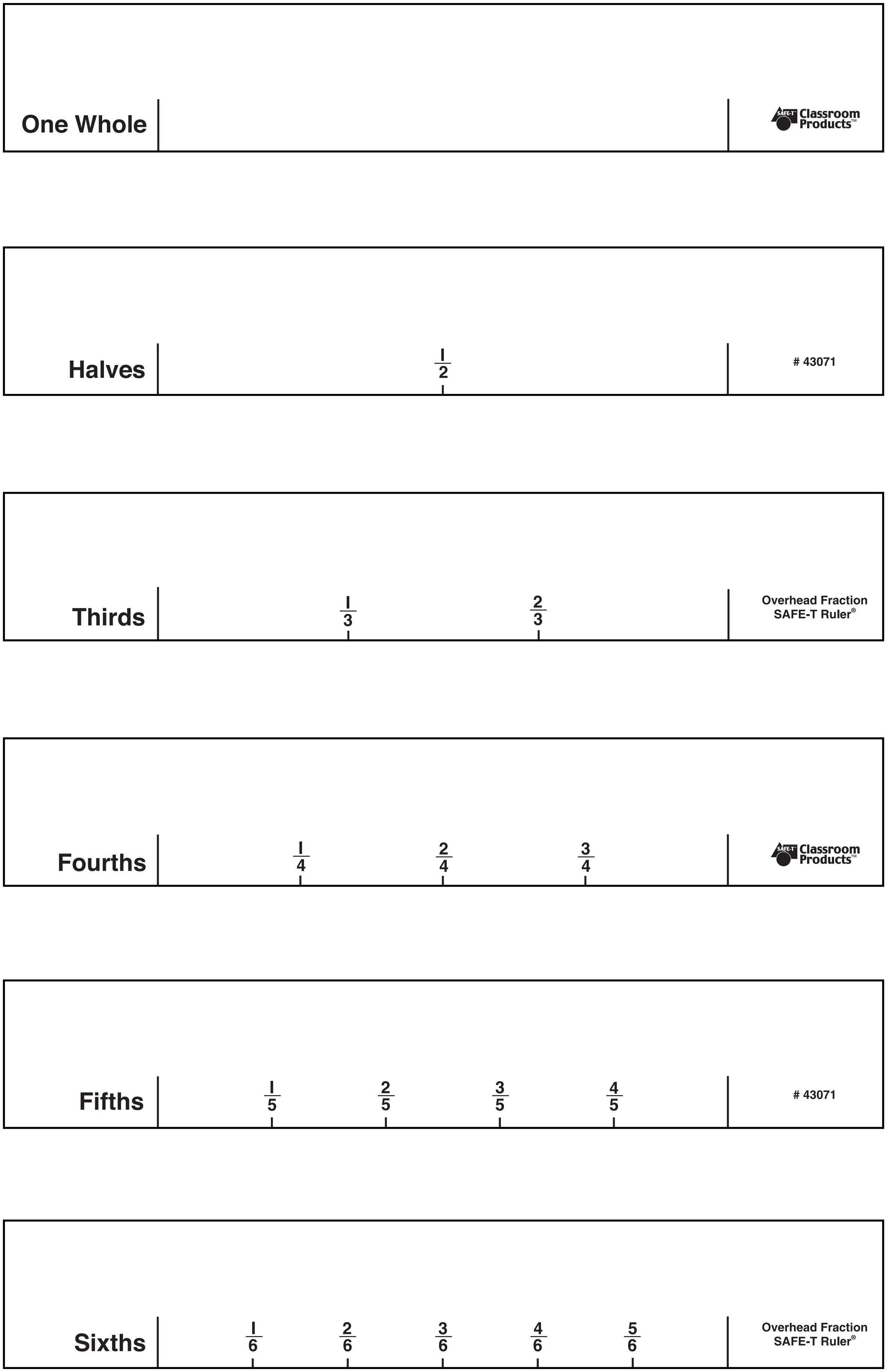


Unit Squares — One Whole





Fraction Ruler



Fraction Ruler

Sevenths | 1/7 | 2/7 | 3/7 | 4/7 | 5/7 | 6/7 | SAFE-T Classroom Products™

Eighths | 1/8 | 2/8 | 3/8 | 4/8 | 5/8 | 6/8 | 7/8 | # 43071

Ninths | 1/9 | 2/9 | 3/9 | 4/9 | 5/9 | 6/9 | 7/9 | 8/9 | Overhead Fraction SAFE-T Ruler®

Tenths | 0.1 1/10 | 0.2 2/10 | 0.3 3/10 | 0.4 4/10 | 0.5 5/10 | 0.6 6/10 | 0.7 7/10 | 0.8 8/10 | 0.9 9/10 | SAFE-T Classroom Products™

Elevenths | 1/11 | 2/11 | 3/11 | 4/11 | 5/11 | 6/11 | 7/11 | 8/11 | 9/11 | 10/11 | # 43071

Twelfths | 1/12 | 2/12 | 3/12 | 4/12 | 5/12 | 6/12 | 7/12 | 8/12 | 9/12 | 10/12 | 11/12 | Overhead Fraction SAFE-T Ruler®